

AP* Calculus Question Type Analysis and Notes

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General note: AP Questions often test several diverse ideas in the same question. The topic names are meant only as a guide and may refer to the form of the question, what it looks like at a glance. Within each type various ideas and concepts may and often are tested.

- GCA = Graphing calculator allowed – yes or no; GCA indicates that the question is in the graphing calculator allowed section of the exam; it does *not* indicate that a graphing calculator should or must be used.
- Comments are highlights, not a complete description
- Some questions listed more than once

Topic 1: Area – Volume Question

Given equations that define a region in the plane students are asked to find its area and the volume of the solid formed when the region is revolved around a line or used as a base of a solid with regular cross-sections. Usually one of the first questions, this standard application of the derivative appears every year and is partly intended to get students started with an easy question.

It is expected that the definite integrals will be evaluated on a calculator. Students should write the definite integral with limits on their paper and put its value after it. It is *not* required to give the antiderivative and if students give an incorrect antiderivative they will lose credit even if the final answer is (somehow) correct.

What students should know how to do:

- Find the intersection(s) of the graphs and use them as limits of integration (calculator equation solving). Write the equation followed by the solution; showing work is not required. Usually no credit until the solution is used in context.
- Find the area of the region between the graph and the x -axis or between two graphs.
- Find the volume when the region is revolved around a line by the disk/washer method. (Shell method is never necessary but is eligible for full credit if properly used).
- Find the volume of a solid with regular cross-sections whose base is the region between the curves.
- Find the equation of a vertical line that divides the region in half (area or volume). This involves setting up and solving an integral equation where the limit is the variable for which the equation is solved.
- For BC only – find the area of a region bounded by polar curves.

Topic 1: Area, volume AB & BC		
Year & Question	GCA	Comments
1998 AB1	Yes	Line divides area or volume into equal parts
1998 BC1	Yes	Line divides volume into equal parts. Suitable for AB
1998 AB2-BC2	Yes	(c) find other lane with same volume
2000 AB1-BC1	Yes	Find intersection, Regular cross-section
2001 AB1	Yes	Find intersection; 2 integrals needed.
2002 AB1- BC1	Yes	Includes max/min
2002 AB1 B	Yes	Volume of revolution and regular cross section
2002 BC3 B	Yes	Curve length Parts a and b suitable for AB
2003 AB1-BC1	Yes	Regular cross-section
2003 BC3	Yes	Area and polar graphs, polar area
2003 AB1 B	Yes	Also write tangent line equation
2003 BC2 B	Yes	Same area with dy and dx , also polar area
2004 AB2-BC2	Yes	Regular cross-section
2004 AB1 B	Yes	Volume of revolution horizontal and vertical lines
2004 AB6-BC6 B	No	Area in terms of parameter.
2004 BC5 B	No	Average value (x^2), Improper integral
2005 AB1-BC1	Yes	Curves define 2 regions.
2005 AB1 B	Yes	Regular cross-section (semi-circles)
2005 BC6 B	No	Improper integrals
2006 AB1-BC1	Yes	Rotations horizontal and vertical lines
2006 AB1-BC1 B	Yes	Area, volume rotation, area.

Topic 2: Particle moving on a line

These questions may give the position equation, the velocity equation or the acceleration equation along with an initial condition. Students may be asked about the motion of the particle: its direction, when it changes direction, its maximum position in one direction etc. Speed, the absolute value of velocity, is also a common topic.

The particle may be a “particle,” a person, car etc. The position, velocity or acceleration may be given as an equation, a graph or a table. There are a lot of different things students may be asked to find. While particles don’t really move in this way, the question is a versatile way to test a variety of calculus concepts.

What students should know how to do (AB and BC):

- Initial value differential equation problems: given the velocity or acceleration with initial condition(s) find the position or velocity.
- Distinguish between position at some time (displacement) and the distance traveled during the time.

- The total distance traveled is the definite integral of the absolute value of the rate of change (velocity): $\int_a^b |v(t)| dt$
- The net distance (displacement) is the definite integral of the rate of change (velocity): $\int_a^b v(t) dt$
- The final position is the initial position plus the definite integral of the rate of change from $x = a$ to $x = t$: $s(t) = s(a) + \int_a^t v(x) dx$ Notice that this is an accumulation function equation.
- Find the speed at a particular time. The speed is the absolute value of the velocity.
- Find average speed, velocity, or acceleration
- Determine if the speed is increasing or decreasing.
 - If at some time, the velocity and acceleration have the *same* sign then the speed is increasing.
 - If they have *different* signs the speed is decreasing.
- Use difference quotient to approximate derivative
- Riemann sum approximations
- Units of measure
- Interpret meaning of integral in context of the problem

Topic 2: Particle Motion on a line for AB and BC		
Year & Question	GCA	Comments
1998 AB3	Yes	Given graph & table: difference quotient, Riemann sum
1999 AB 1	Yes	Given v : analyze motion, find a , s and total distance
2000 AB2-BC2	Yes	Given graph & equation
2001 AB3-BC3	Yes	Given a graph: analyze v , max/min, FTC
2002 AB3	Yes	Given v equation: Speed/velocity, Speed increasing?
2002 AB3 B	Yes	Given v equation: Sketch graph, analyze motion
2003 AB2	Yes	Given v graph: Speed increasing? Analyze motion
2003 AB4 B	No	Given v graph: Speed increasing? Analyze motion
2004 AB3	Yes	Given v graph: Speed increasing? Analyze motion
2004 AB3-BC3 B	Yes	Given table of v : explain integral, MVT, average v
2005 AB5-BC5	No	Given v graph: distance, a , average rate of change
2005 AB3 B	Yes	Given v equation: analyze motion.
2006 AB4	No	Table & equation, average, Riemann sum,
2006 AB6 B	No	From table, distance, FTC, analyze motion

Topic 3: Particle moving on a plane for BC (parametric/vector question).

On the BC exam particles often move in the plane. Their position is defined by two parametric equations $x = x(t)$ and $y = y(t)$. The velocity is the vector $(x'(t), y'(t))$ and

the acceleration is the vector $(x''(t), y''(t))$. Any of these three may be given with initial conditions(s) and student may be asked to find the others.

What students should know how to do:

- Initial value differential equation problems – given the velocity or acceleration with initial conditions, find the position and/or velocity.
- Use the definite integral for arc length to find the distance traveled.
- Find the speed at time t : speed = $\sqrt{(x'(t))^2 + (y'(t))^2}$
- Vectors are given in ordered pair form; answers may be in ordered pairs form or \vec{i} - \vec{j} form.

Topic 3: Particle motion in the plane; parametric equation, vectors for BC		
Year & Question	GCA	Comments
1998 BC6	Yes	Rectangular and parametric together, position, speed.
1999 BC1	Yes	Given position: graph, max/min, acceleration, speed
2000 BC4	No	Given v : acceleration, position, slope, limit as $t \rightarrow \infty$
2001 BC1	No	Given v : Distance, position by accumulation, speed
2002 BC3	Yes	Slopes, distance, max/min
2002 BC1 B	Yes	Given position: draw graph, speed, distance
2003 BC2	Yes	Given x' but not y' : an atypical problem
2003 BC4 B	No	Given position: when moving horizontally and vertically
2004 BC3	Yes	Given position: tangent lines, speed
2004 BC1 B	Yes	Given velocity vector
2005 BC1 B	Yes	Equation of tangent line; when is particle at rest?
2006 AB4-BC4	No	Table & equation, average, Riemann sum,
2006 BC3	Yes	Parametric velocity, tangent line, limits, improper integral
2006 BC2 B	Yes	Tangent line, acceleration speed, distance traveled, explain.

Topic 4: Interpreting Graphs

There are a variety of question types here. Students may be given an equation and asked for the location of extreme values, intervals where the function is increasing or decreasing, concavity, etc. Students may be given the graph of the derivative and asked the same kinds of things. They may be asked to find the value of the integral given the graph but no equation.

What students should know how to do:

- Reading information about the function from the graph of the derivative. This may be approached as a derivative techniques or antiderivative techniques.
- Find and justify extreme values (1st DT, 2nd DT).
- Find and justify points of inflection.
- Write an equation of tangent line
- Evaluate Riemann sums from graphs only.
- FTC from graph areas.
- “Family” of functions”: functions with a parameter;
- Functions defined by other functions

Topic 4: Information from graph or about the graph, family of functions, f, f', f''		
Year & Question	GCA	Comments
1998 AB2- BC2	Yes	Limits, max/min, “family”
1999 AB4	Yes	From equation, 2-DT, function defined by other function
1999 AB5-BC5	Yes	From graph: Max/min, inc/dec, tangent line,
2000 AB3	Yes	From graph: f, f', f'' , inc/dec, POI
2001 AB3-BC3	Yes	Particle motion (q.v.) setting
2001 AB4-BC4	No	From derivative equation: Max/min, tangent line, concavity
2002 AB1 BC1 (c)	Yes	Includes max/min area/volume
2002 AB4-BC4	No	Given f' graph: inc/dec, concavity, draw graph f
2002 BC5	No	Slope field w/ draw solution, Euler, max/min
2002 AB2 B	Yes	Given f' : Max/min, accumulation

2002 AB4-BC4 B	No	Given graph f : values, inc/dec, concavity, Trap rule.
2003 AB3	Yes	Given table and graph: Diff quotient, Riemann sum, max/min, average value, accumulation
2003 AB4-BC4	No	Given graph f' : inc/dec, tangent line, FTC by area.
2003 AB5 B	No	Given graph f : values, average values, MVT, POI
2004 AB5	No	Given graph f : values, FTC, max/min, POI
2004 AB2 B	Yes	Accumulation setting, max/min, inc/dec.
2004 AB4-BC4 B	No	Given graph f' :inc/dec, product rule
2005 AB4	No	Tabular information about f, f', f'' , FTC, max/min, draw graph, POI
2005 AB4-BC4 B	No	Given graph f' : values, FTC from graph area, inc/dec.
2006 AB3	Yes	Periodic function, accumulation, tangent line
2006 AB 2 B	Yes	Concavity, max/min, tangent line
2006 AB4-BC4 B	Yes	From graph, derivative, area

Topic 5: Accumulation & Rates

The integral of a rate of change gives the amount of change. The general form of the equation is $F(x) = F(x_0) + \int_{x_0}^x f(t) dt$ where F is an antiderivative of f , $x = x_0$ is the initial time, and $F(x_0)$ is the initial value. Since this is one of the main interpretations of the definite integral the concept may come up in a variety of situations.

What students should know how to do:

- Understand the question. It is often not necessary to do as much computation as it seems at first.
- The FTC may help differentiating F .
- Often these problems contain a lot of writing; be ready to read and apply.
- Explain the meaning of a derivative or definite integral or its value in terms of the context of the problem.
- In-out problems: 2 rates of change work together
- Max/min and inc/dec analysis.

Topic 5: Accumulation & Rates		
Year & Question	GCA	Comments
1998 AB5-BC5	Yes	Draw graph, average value, accumulation
1999 AB3-BC3	Yes	From table: Riemann sum, Rolle's theorem – MVT, average value
2000 AB4	No	2 solution methods: accumulation, max/min.
2002 AB2-BC2	Yes	“Amusement Park:” In-out, amounts, max/min, values
2002 AB2-BC2 B	Yes	Pollutants: max/min, amounts, accumulation
2003 AB3	Yes	From graph and table: difference quotient, max/min, Riemann sum, interpret integrals.
2003 AB2 B	Yes	Heating oil: In-out, inc/dec, accumulation, max/min
2004 AB1-BC1	Yes	Traffic flow: average value, average rate of change, accumulation, inc/dec.
2004 AB2 B	Yes	Mosquitoes: values, inc/dec, accumulation, max/min
2005 AB2	Yes	“Sandy Beach:” In-out, amounts, max/min, FTC
2005 AB2- BC2 B	Yes	Water tank: In-out, amount, max/min, inc/dec
2006 AB2-BC2	Yes	“Thomasville:” Average value and application

Topic 6: Problems from information given in tables

Tables may be used to test a variety of ideas in calculus.

What students should be able to do:

- Approximate the derivative using a difference quotient.
- Use Riemann sums (left, right, midpoint) or the Trapezoidal Rule to approximate the value of a definite integral using values in the table (typically with uneven subintervals).
- Average value and the MVT may appear

Do not: Use a calculator to find a regression equation and then use that to answer parts of the question. (While finding them is perfectly good mathematics, regression equations are not one of the four things students may do with their calculator.)

Topic 6: Table		
Year & Question	GCA	Comments
1998 AB3	Yes	Given graph & table: difference quotient, Riemann sum
1999 AB3-BC3	Yes	From table: Riemann sum, Rolle – MVT, average value
2001 AB2-BC2	Yes	Difference quotient, Trap rule, explain, average value
2002 AB6	No	Definite integral, tangent line, MVT, interpret, limits
2003 AB3	Yes	From graph and table: difference quotient, max/min, Riemann sum, interpret integrals.
2003 AB3 B	Yes	Average value, Riemann sum, explain, MVT
2004 AB3-BC3 B	Yes	Given table of v : explain integral, MVT, average v
2005 AB3-BC3	Yes	Tabular information, FTC, max/min, draw graph, POI
2005 AB4	No	Tabular information about f, f', f'' , FTC, max/min, draw graph, POI
2006 AB4	No	Table & equation, particle motion: average, Riemann sum,
2006 AB6 B	No	From table, distance, FTC, analyze motion

Topic 7: Differential Equation Questions

Differential equations are tested every year. The actual solving of the differential equation is usually the main part of the problem, but it is accompanied by a question about its slope field or a tangent line approximation of some sort. BC students may also be asked to approximate using Euler’s Method. Large parts of the BC questions are often suitable for AB students and contribute to the AB subscore of the BC exam..

What students should be able to do:

- Find the *general solution* of a differential equation using the method of separation of variables (this is the *only* method tested).
- Find a *particular solution* using the initial condition to evaluate the constant of integration – initial value problem (IVP)
- Understand that proposed solution of a differential equation is a function (not a number) and if it and its derivative is substituted into the given differential

equation the resulting equation is true. This may be part of doing the problem even if solving the differential equation is not required (see 2002 BC 5(c))

- Growth-decay problems.
- Draw a slope field by hand.
- Sketch a *particular solution* on a (given) slope field.
- Interpret a slope field.
- For BC only: Use Euler’s Method to approximate a solution.
- For BC only: use the method of partial fractions to find the antiderivative after separating the variables.
- For BC only: understand the logistic growth model, its asymptotes, meaning, etc.

Topic 7: Differential Equations		
Year & Question	GCA	Comments
1998 AB4	Yes	IVP, equation of tangent line
1998 BC4	Yes	IVP field, Euler, solve
1999 BC6	Yes	Tangent line with approximation, Euler, FTC with “impossible integrand” in (c)
2000 AB6	No	IVP, find domain and range of solution.
2000 BC6	No	Slope field with analysis, IVP, range
2001 AB6	No	Second derivative by implicit differentiation of y' , IVP
2001 BC5	No	Improper integral, Euler, IVP
2002 BC5	No	Draw solution on given slope field, Euler, max/min, check solution, 2-DT,
2002 AB5-BC5 B	No	2-DT, IVP
2003 AB5-BC5	No	Related Rate, IVP
2003 BC6 (c)	No	From power series check solution of differential equation
2003 AB 6 B	No	Second derivative by implicit differentiation of y' , IVP
2004 AB6	No	Draw slope field, generalize, IVP
2004 BC5	No	Logistics Diff Eq, limits from y' and y , interpret, IVP
2004 AB5 B	No	Draw slope field, generalize, IVP

2005 AB6	No	Draw slope field, tangent line approximation, IVP
2005 BC4	No	Draw slope field and sketch solution on it; max/min, Euler, implicit differentiation for y' and analyze.
2005 AB6 B	No	Draw slope field, tangent line, solve
2006 AB 5	No	Draw slope field, solve, domain
2006 BC 5	No	Implicit y'' , write Taylor Poly, Euler
2006 AB 5 B	No	Draw slope field, solve IVP
2006 BC5 B	No	Solve IVP, Analyze logistic DE w/o solving.

Topic 8: Power Series (BC only)

Since some graphing calculator can produce Taylor Polynomials, this question appears on the no calculator allowed section. (Questions from before the FR sections was split (1995 – 1999) do not have anything a calculator *could* do. They are interesting and clever.)

What students should be able to do:

- Find the Taylor (or Maclaurin) polynomial or series for a given function – usually 4 terms and the general term). This may be done by finding the various derivatives, or any other method such as substitution into a know series, long division, the formula for the sum of an infinite geometric series, integration, differentiation, etc.
- Know from memory the Maclaurin series for $\sin(x)$, $\cos(x)$, e^x , and $\frac{1}{1-x}$
- Find related series by substitution, differentiation, integration or by adapting one of those above.
- Find the radius of convergence (usually by using the Ratio test, or from a geometric series).
- Find the interval of convergence using the radius and checking the endpoints separately.
- Handle geometric series.
- Use the convergence tests.
- Find a high-order derivative from the coefficient of a term.
- Estimate the error bound of a Taylor or Maclaurin polynomial by using *alternating series error bound* or the *Lagrange error bound*.
- *Do not* claim that a function is equal to (=) its Taylor or Maclaurin polynomial; it is only approximately equal (\approx).

Topic 8: Power Series BC only		
Year & Question	GCA	Comments
1998 BC3	Yes	Write Taylor, new series by substitution, and by integration, analyze. CAS no help
1999 BC4	Yes	Write Taylor series, Lagrange error bound, max/min CAS no help
2000 BC3	No	Write series, radius of convergence, Alternating series error bound
2001 BC6	No	Interval of convergence, limit, integrate, Geometric series sum
2002 BC6	No	Interval of convergence, differentiate.
2002 BC6	No	Substitute, series convergence
2003 BC6	No	Max/min 2-DT, Alternating series error; check solution of differential equation
2003 BC6 B	No	Write series, radius of convergence, interval of convergence
2004 BC6	No	Write series, Find high-order coefficient, Lagrange error bound, integrate
2004 BC2 B	Yes	Find derivatives, analyze, Lagrange error bound
2005 BC6	No	Write series, find general coefficient, interval of convergence
2005 BC3 B	Yes	Given $f^{(n)}$, max/min 2-DT, write series, find radius of convergence.
2006 BC 5	No	Implicit y'' , write Taylor Poly, Euler
2006 BC 6	No	Interval w/ endpoints, y' and y'' from coefficients max/min
2006 BC6 B	No	Differentiate, integrate, alternating series error bound.

Topic 9: Other Topics tested occasionally

Implicitly defined relations and implicit differentiation

These questions may ask students to find the first or second derivative of an implicitly defined relation. Often the derivative is given and students are required to show that it is

correct. (This is because without the correct derivative the rest of the question cannot be done.)

What students should know how to do:

- Know how to find the first derivative of an implicit relation using the product rule, quotient rule, the chain rule ,etc.
- Know how to find the second derivative, including substituting for the first derivative.
- Know how to evaluate the first and second derivative by substituting both coordinates of the point,
- Analyze the derivative to determine where the relation has horizontal and/or vertical tangents.
- Work with lines tangent to the relation.

Related Rates

Derivatives are rates and when more than one variable is involved the relationships among the rates can be found by differentiating with respect to time. The time variable may not appear in the equations.

What students should know how to do:

- Know how to differentiate using the product, quotient and chain rules.
- Set up and solve related rate problems.
- Interpret the answer.

Topic 9 Miscellaneous		
Year & Question	GCA	Comments
1998 AB6	Yes	Implicit diff, analyze implicit relation, tangent lines
1999 AB6	Yes	Related Rate
2000 AB5-BC5	No	Implicit diff, analyze implicit relation, tangent lines
2001 AB5	No	Unique graph problem, FTC
2002 AB5	No	Related Rate, units of measure
2002 AB6 B	No	Related Rate
2003 AB5-BC5	No	Related Rate w/ solve differential equation IVP
2003 AB6	No	Continuity, average value, parameters

2003 BC3	Yes	Area and polar graphs, polar area
B 2003 BC3	Yes	Includes polar area
2004 AB4-BC4	No	Implicit diff, analyze, second derivative
2004 AB6-BC6 B	No	Use integral with parameter
2005 BC2	Yes	Polar Graph, max/min, interpret $dr/d\theta$
2005 AB5-BC5 B	No	Implicit diff, analyze implicit relation
2006 AB6	No	Differentiation without functions.
2006 BC3	Yes	Parametric velocity, tangent line, limits, improper integral
2006 BC 5	No	Implicit y'' , write Taylor Poly, Euler
2006 AB3-BC3 B	Yes	Graph investigation

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